Pergamon

Int. J. Solids Structures Vol. 31, No. 24, pp. 3457–3473, 1994

0020–7683/94 $7.00 + .00

0020–7683(94)00104–9

A METHOD OF STRAIN AND STRESS ANALYSIS OF COMPOSITES FOR NONLINEAR STRAIN DISTRIBUTION CASE

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(*Received* 19 *November* 1993; *in revised form* 2 *March* 1994)

Abstract—In this study, a method of strain and stress analysis is developed to determine the stress and strain fields in fibers and matrix of a laminated composite for the cases of nonlinear strain and stress distributions with high gradients. The strain values calculated by using a classical method are considered as input, and the strain fields at each layer are determined. These strain fields are then used as input to express the strains in fibers and matrix. In these processes the heterogeneity effects, which are caused by the existence of different materials in a composite, are taken into account. By the summation of input strains and the heterogeneity strains, the strains in the fibers and the matrix are calculated. By using the fiber and matrix strains in constitutive equations the stresses in fibers and matrix are obtained. The non-classical method is applied to a sample problem and numerical results are presented as well as a comparison of the layer stresses with the results from MSC/Nastran finite element software composite material solution.

NOTATION

A_{ij}	stiffness matrix elements for the laminate
c_f	in-fiber heterogeneity variation factor
c_{ij}^m	in-matrix heterogeneity variation factor
e_{ij}	far-field strains
e_{ij}^n	average layer strain components
$\bar{e}_{ij}^{nf}$	average fiber strains
$\bar{e}_{ij}^{nm}$	average matrix strains
$\bar{e}_{ij}^{nf^+}, \bar{e}_{ij}^{nf^-}$	average strains in fibers due to the right and left adjacent matrix regions, respectively, in layer coordinates
$\bar{e}_{ij}^{nm^+}, \bar{e}_{ij}^{nm^-}$	average strains in matrix due to the right and left adjacent fibers, respectively, in layer coordinates
f_{ij}^n	adjacent material region effects on fibers
$f_{ij}^{n^+}, f_{ij}^{n^-}$	right and left heterogeneity effects on fibers, respectively
m_{ij}^n	adjacent material region effects on matrix regions
$m_{ij}^{n^+}, m_{ij}^{n^-}$	right and left heterogeneity effects on matrix, respectively
N	total number of laminae in a laminate
$\bar{Q}_{ij}^n$	transformed stiffness matrix elements for each layer
R_i	unit composite dimensions
R_f	radius of a fiber bundle
R_{fi}, R_{mi}	unit composite fiber and matrix dimensions, respectively
r_n	layer inner characteristic length
S_{ij}^{nf}	fiber heterogeneity strain component
S_{ij}^{nm}	matrix heterogeneity strain component
$s_{ij}^{nf^+}, s_{ij}^{nf^-}$	right and left heterogeneity effect sign term for the fibers
$s_{ij}^{nm^+}, s_{ij}^{nm^-}$	right and left heterogeneity effect sign term for the matrix
x_{0i}^f, x_{0i}^m	fiber and matrix central coordinates, respectively, in global coordinate system
x_{0i}^n	nth layer center coordinates
δ_{ij}^n	adjacent layer effects
ε_{ij}^n	in-layer strain components
$\bar{\varepsilon}_{ij}^n$	transformed in-layer strain components
ε_{ij}^{nf}	fiber strain components in global coordinates
ε_{ij}^{nm}	matrix strain components in global coordinates
$\bar{\varepsilon}_{ij}^{nf}$	transformed fiber strain components
$\bar{\varepsilon}_{ij}^{nm}$	transformed matrix strain components
κ_{ij}^n	layer heterogeneity strains
λ_f	fiber Lamé constant
λ_m	matrix Lamé constant
μ_f	fiber Lamé constant
μ_m	matrix Lamé constant
θ_n	angle between the global x_1-axis and the fiber direction

τ_{ij}^{nf} fiber stress fields in global coordinates
τ_{ij}^{nm} matrix stress fields in global coordinates.

1. INTRODUCTION

Laminated composites consist of several numbers of layers, each of which may contain continuous, unidirectional reinforcing fibers. An important step in the understanding of the mechanical behavior and the prediction of the failure of a composite body is the determination of the strain and stress fields in each layer, moreover in the fibers and matrix inside each lamina, for a general case of nonlinear strain and stress distributions with high gradients.

In the analysis of composites, laminated plate or shell theories are generally used. These theories assume linear variations of the in-plane displacement components through the thickness. For relatively thick laminates the assumption of linear displacement can lead to considerable inaccuracy. This has been shown in the work of Pagano and Hatfield (1972), and Pagano (1970) who investigated the limitations of classical laminated plate theory.

In most cases the main step towards the strain and stress analyses is the determination of the effective stiffness coefficients for the overall composite material. The essential quantities sought from micromechanics formulations are the effective moduli defined as the constitutive coefficients relating the volume averages of the strain components. A number of approaches have been devised for calculating the effective material moduli of composites. Hashin (1983) has presented a survey of these analyses from the applied mechanics and engineering science point of view. For a two-dimensional, bimaterial composite body under elastic deformation, Ardıç *et al.* (1989) developed a method to determine separate expressions of stress fields for each material in the body. Ardıç *et al.* (1990) extended this study to the three-dimensional case for a unidirectional laminate. By using this non-classical method strain and stress fields in the fibers and matrix can be calculated. This method is similar to the nonlocal elasticity in principle; by considering the long-range effects caused by the inhomogeneity on a micro-structural scale, the interactions between the point of consideration and the other material regions are expressed. In this process material moduli of the fibers and the matrix are used so that there is no need to calculate effective moduli of the laminate. The method developed by Ardıç *et al.* (1989) and (1990), and this study considers the heterogeneity related long-range interactions among different material regions and does not deal with the determination of effective moduli. Futhermore, the method can be used to calculate strains and stresses in the fibers and the matrix. Considering all these, it is clear that the present study is a different analysis method than the classical plate and shell theories. Therefore, this method is called "non-classical" throughout this study. Ardıç and Santare (1991) extended the method to a laminated composite section case, but in that study only two different kinds of alternating layers are considered, for example $[0^\circ/90^\circ]_n$; in reality, a laminated composite body may consist of several kinds of laminae, such as $[0^\circ/\pm 45^\circ/90^\circ]_{2s}$. Additionally, the loading and notches in the material may cause very high stress and strain gradients with nonlinear distributions. For this reason, the non-classical method should be developed further for a general case which considers a laminated composite having layers with different material properties and nonlinear strain and stress distributions with high gradients for realistic applications.

In this study, the objective is to extend the non-classical method to the most general case of multi-layer laminate with high gradient nonlinear stress and strain distributions. Section 2 of this study shows the determination of the layer strains for the above mentioned general case. In Section 3, the expressions for the strain and stress fields in fibers and matrix are obtained. A sample problem is solved in Section 4 which also contains comparisons and discussions. Section 5 offers an outline and concluding remarks.

2. STRAINS IN LAYERS

In this study, the strain and stress fields in fibers and matrix in a laminated composite section are to be determined. In a former study by Ardıç and Santare (1991), the laminae

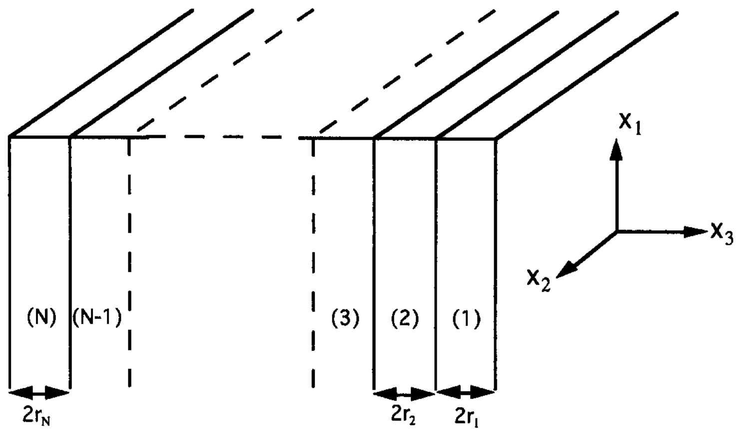

Fig. 1. Structure of the laminated composite section.

are assumed to be alternating and there are just two different kinds of layers which may have different fiber orientations, volume fractions, fiber and matrix materials. If there were N different kinds of layers, the method for two alternating layers would no longer be valid. But still the average of the strains in each layer should satisfy the input far-field strains. The strain components, calculated by classical methods like plate and shell theories, finite elements method, or measured, are used as input to the analysis method presented in this study, and these input strain components are called "far-field" strains. In this case the strains including the far-field strains are functions of position due to the nonlinear strain distribution which may have a high gradient.

In Fig. 1, the structure of a laminated composite section, containing N number of layers, can be seen. Since the strain and stress fields in the fibers and matrix are to be determined, first the non-classical elasticity analysis is used to calculate the strains in each layer on the assumption that each layer is homogeneous and orthotropic. These strains are then used as input strains to determine the strains and stresses in the fibers and matrix. The strains in each layer are called the "in-layer" strains throughout this paper as was done by Ardıç and Santare (1991).

To determine these in-layer strains, at the beginning a simple classical elasticity analysis can be used. In the study of Ardıç and Santare (1991), the non-classical elasticity analysis was used. The idea in the present study is the same with the exception that each layer may have different material properties, the strains are strongly dependent on position, (x_i), the expressions of the heterogeneity effects are more complicated. The far-field strains for the body are considered to be known and are used as input. The formulation of this elasticity problem follows that by Ardıç *et al.* (1989) but here the problem is three-dimensional and the layers are anisotropic. First, to determine the effects of two adjacent layers on each other, a simple elasticity analysis is performed. The geometry of the problem is also shown in Fig. 1. The initially calculated or measured strain components, $e_{ij}(x_k)$ are assumed to be known. The interface and the averaging conditions for the system shown in Fig. 1 are: (i) laminae are perfectly bonded to each other; (ii) the averages of the strain components in all laminae are equal to the corresponding far-field strain components for the laminate; and (iii) tractions are equal at the interfaces.

The far-field strain components are assumed to be uniform inside each layer for simplicity. Therefore, the x_3 coordinate dependence of the strains is reduced to layer number dependence, because the strain values at the center of each lamina are used in the expressions. Following this statement, the average strains in the layers are found by satisfying the conditions above as follows: for an arbitrary layer "n"

$$e_{11}^n = e_{11}(x_{03}^n), \quad e_{22}^n = e_{22}(x_{03}^n),$$

$$e_{33}^n = \frac{(A_{31}-\bar{Q}_{31}^n)e_{11}(x_{03}^n)+(A_{23}-\bar{Q}_{23}^n)e_{22}(x_{03}^n)+A_{33}e_{33}(x_{03}^n)}{\bar{Q}_{33}^n},$$

$$e_{23}^{n}=\frac{A_{44}}{\bar{Q}_{44}^{n}}e_{23}(x_{03}^{n}),\quad e_{31}^{n}=\frac{A_{55}}{\bar{Q}_{55}^{n}}e_{31}(x_{03}^{n}),\quad e_{12}^{n}=e_{12}(x_{03}^{n}),\tag{1}$$

where "n" is the layer identification number starting from one boundary of the laminate in the x_3-direction, $n = 1, 2, 3, \ldots, N$ and "N" is the total number of laminae. $e_{ij}(x_{03}^{n})$ are the values of far-field strains at the centers of laminae "n". $\bar{Q}_{ij}^{n}$ are the transformed stiffness matrix elements for each layer that are calculated from the expressions given by Vinson and Sierakowski (1987). The average of these stiffness coefficients is the stiffness matrix elements for the laminate; these coefficients can be expressed as

$$A_{ij}=\frac{\sum_{n=1}^{N}\bar{Q}_{ij}^{n}r_{n}}{\sum_{n=1}^{N}r_{n}}.\tag{2}$$

Up to this point the strain components in the layers are determined by using a classical elasticity analysis, but actually the strains in each layer are affected by the other layers similar to the nonlocal elasticity theory. These effects occur only in x_3-direction for the structure in Fig. 1, because these interactions that are called heterogeneity effects are due to the material inhomogeneity. By summing the heterogeneity effects and the far-field strains, in-layer strains are determined. Derivations and explanations about the heterogeneity effects are widely presented in by Ardıç *et al.* (1989) and Ardıç and Santare (1991). Therefore, the in-layer strain components can be presented as follows:

$$\begin{aligned}&\varepsilon_{11}^{n}=e_{11},\quad \varepsilon_{22}^{n}=e_{22},\quad \varepsilon_{33}^{n}=e_{33}+\kappa_{33}^{n},\\&\varepsilon_{23}^{n}=e_{23}+\kappa_{23}^{n},\quad \varepsilon_{31}^{n}=e_{31}+\kappa_{31}^{n},\quad \varepsilon_{12}^{n}=e_{12},\end{aligned}\tag{3}$$

where

$$\kappa_{ij}^{n}=\delta_{ij}^{n}-\frac{1}{2}\sum_{m=1}^{n-1}\left(\prod_{k=m}^{n}\delta_{ij}^{k}\right)-\frac{1}{2}\sum_{m=n+1}^{N}\left(\prod_{k=n}^{m}\delta_{ij}^{k}\right),\tag{4}$$

$$\delta_{ij}^{n}=(e_{ij}^{n}-e_{ij}).\tag{5}$$

The in-layer strains in each layer have therefore been determined. These strain components are written relative to the global coordinate system, but it is convenient to express the strains such that the reinforcement direction is defined as the x_1-direction. For this reason the in-layer coordinates are rotated around the x_3-axis so that the in-layer x_1'-direction corresponds to fiber direction. The in-layer strains are therefore found by applying a coordinate transformation to the strain components expressed by eqns (3) such that

$$\begin{aligned}\bar{\varepsilon}_{11}^{n}&=\cos^{2}(\theta_{n})\varepsilon_{11}^{n}+\sin^{2}(\theta_{n})\varepsilon_{22}^{n}+2\cos(\theta_{n})\sin(\theta_{n})\varepsilon_{12}^{n},\\\bar{\varepsilon}_{22}^{n}&=\sin^{2}(\theta_{n})\varepsilon_{11}^{n}+\cos^{2}(\theta_{n})\varepsilon_{22}^{n}-2\cos(\theta_{n})\sin(\theta_{n})\varepsilon_{12}^{n},\\\bar{\varepsilon}_{33}^{n}&=\varepsilon_{33}^{n},\\\bar{\varepsilon}_{23}^{n}&=\cos(\theta_{n})\varepsilon_{23}^{n}-\sin(\theta_{n})\varepsilon_{31}^{n},\\\bar{\varepsilon}_{31}^{n}&=\cos(\theta_{n})\varepsilon_{31}^{n}+\sin(\theta_{n})\varepsilon_{23}^{n},\\\bar{\varepsilon}_{12}^{n}&=\cos(\theta_{n})\sin(\theta_{n})(\varepsilon_{22}^{n}-\varepsilon_{11}^{n})+\cos(2\theta_{n})\varepsilon_{12}^{n},\end{aligned}\tag{6}$$

where θ_n is the angle between the global x_1-axis and the fiber direction for the layer "n".

By using eqns (1)–(6) the in-layer strains for each lamina are calculated. At this point by using these in-layer strains as input, the strain and stress fields in fibers and matrix of each lamina can be calculated separately.

3. STRESSES IN FIBERS AND MATRIX

In this section, the in-layer strain components expressed by eqns (3)–(6) are considered as the input far-field strains to determine the strains and stresses in the fibers and matrix regions. The method used to determine the stress and strain fields in fibers and matrix, is similar to the method developed by Ardıç *et al.* (1990), although this method cannot be used for a general case, the procedure is similar. The average strains in the fibers and the matrix are calculated by a unit volume approach. The unit volume considered is shown in Fig. 2. The following assumptions have been made in the calculation of the average strains: (i) the strain components along the fiber direction are the same for the fiber and the matrix, and they are equal to the corresponding in-layer strain components; (ii) the averages of the other strain components over the unit volume are equal to the corresponding input in-layer strain components; (iii) tractions are equal at the interfaces, and it is assumed that the in-layer strains are uniform inside each material region; and (iv) in addition, the form of the variation of the heterogeneity effects inside each material region is assumed such that the above conditions are satisfied.

At this point a simple elasticity analysis is applied to find the average strains in the fibers and the matrix in a unit composite shown in Fig. 2. The derivations of the expressions for three-dimensional, unidirectional composites with linear or uniform strain fields are presented by Ardıç *et al.* (1990). But in this case, since the strain variations can be nonlinear with high gradients, the averaging condition "*ii*" is used in a different way where the displacements should be calculated by integrating the strains. The average strains in the fibers and the matrix are represented by $\bar{e}_{ij}^{n}$ and $\bar{e}_{ij}^{nm}$, respectively, in this study. Also for this case a unit volume approach is used. The strains at the material region of consideration is calculated separately by considering the left and the right adjacent material regions. The strain components at the nth layer due to the existence of left adjacent material region are represented with $e_{ij}^{nf^-}$ for fibers, $e_{ij}^{nm^-}$ for matrix, and of right adjacent material are represented with $e_{ij}^{nf^+}$ for fibers, $e_{ij}^{nm^+}$ for matrix. These strains include the effect of adjacent fiber or matrix on considered matrix or fiber, and they can be expressed as follows.

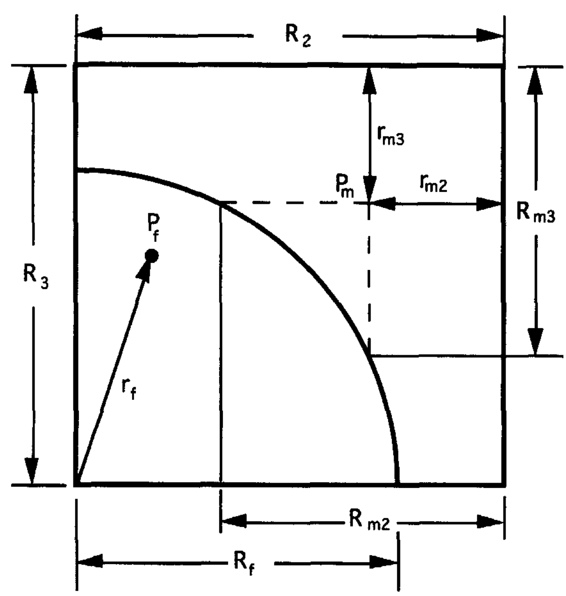

Fig. 2. Fiber and matrix characteristic dimensions.

For the fibers,

$$\bar{e}_{11}^{nf^+} = \bar{e}_{11}^{nf^-} = \bar{\varepsilon}_{11}^n, \tag{7a}$$

$$\bar{e}_{22}^{nf^+} = \left\{(\lambda_m-\lambda_f)R_{m2}\bar{\varepsilon}_{11}^n + (2\mu_m+\lambda_m)\int_{x_{02}^f}^{x_{02}^f+R_2}\bar{\varepsilon}_{22}^n\,\mathrm{d}x_2 + \lambda_m\frac{R_{m2}}{R_{m3}}\int_{x_{03}^f}^{x_{03}^f+R_3}\bar{\varepsilon}_{33}^n\,\mathrm{d}x_3 - \left(\lambda_m\frac{R_{f3}}{R_{m3}}+\lambda_f\right)R_{m2}\bar{e}_{33}^{nf^+}\right\}\Big/\{(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}\}, \tag{7b}$$

$$\bar{e}_{22}^{nf^-} = \left\{(\lambda_m-\lambda_f)R_{m2}\bar{\varepsilon}_{11}^n + (2\mu_m+\lambda_m)\int_{x_{02}^f-R_2}^{x_{02}^f}\bar{\varepsilon}_{22}^n\,\mathrm{d}x_2 + \lambda_m\frac{R_{m2}}{R_{m3}}\int_{x_{03}^f-R_3}^{x_{03}^f}\bar{\varepsilon}_{33}^n\,\mathrm{d}x_3 - \left(\lambda_m\frac{R_{f3}}{R_{m3}}+\lambda_f\right)R_{m2}\bar{e}_{33}^{nf^-}\right\}\Big/\{(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}\}, \tag{7c}$$

$$\bar{e}_{33}^{nf^+} = \frac{1}{\delta_0}\left\{A_1\bar{\varepsilon}_{11}^n + A_2\int_{x_{02}^f}^{x_{02}^f+R_2}\bar{\varepsilon}_{22}^n\,\mathrm{d}x_2 + A_3\int_{x_{03}^f}^{x_{03}^f+R_3}\bar{\varepsilon}_{33}^n\,\mathrm{d}x_3\right\}, \tag{7d}$$

$$\bar{e}_{33}^{nf^-} = \frac{1}{\delta_0}\left\{A_1\bar{\varepsilon}_{11}^n + A_2\int_{x_{02}^f-R_2}^{x_{02}^f}\bar{\varepsilon}_{22}^n\,\mathrm{d}x_2 + A_3\int_{x_{03}^f-R_3}^{x_{03}^f}\bar{\varepsilon}_{33}^n\,\mathrm{d}x_3\right\}, \tag{7e}$$

$$\bar{e}_{23}^{nf^+} = \frac{\mu_m\left\{\int_{x_{02}^f}^{x_{02}^f+R_2}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_2 + \frac{1}{2}\int_{x_{03}^f-R_3}^{x_{03}^f+R_3}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_3\right\}}{2\left\{\mu_m R_f + \mu_f\left(\frac{R_2}{R_3}-R_f\right)\right\}}, \tag{7f}$$

$$\bar{e}_{23}^{nf^-} = \frac{\mu_m\left\{\int_{x_{02}^f-R_2}^{x_{02}^f}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_2 + \frac{1}{2}\int_{x_{03}^f-R_3}^{x_{03}^f+R_3}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_3\right\}}{2\left\{\mu_m R_f + \mu_f\left(\frac{R_2}{R_3}-R_f\right)\right\}}, \tag{7g}$$

$$\bar{e}_{31}^{nf^+} = \frac{\mu_m\int_{x_{03}^f}^{x_{03}^f+R_3}\bar{\varepsilon}_{31}^n\,\mathrm{d}x_3}{\mu_m R_{f3}+\mu_f R_{m3}}, \quad \bar{e}_{31}^{nf^-} = \frac{\mu_m\int_{x_{03}^f-R_3}^{x_{03}^f}\bar{\varepsilon}_{31}^n\,\mathrm{d}x_3}{\mu_m R_{f3}+\mu_f R_{m3}}, \tag{7h}$$

$$\bar{e}_{12}^{nf^+} = \frac{\mu_m\int_{x_{02}^f}^{x_{02}^f+R_2}\bar{\varepsilon}_{12}^n\,\mathrm{d}x_2}{\mu_m R_{f2}+\mu_f R_{m2}}, \quad \bar{e}_{12}^{nf^-} = \frac{\mu_m\int_{x_{02}^f-R_2}^{x_{02}^f}\bar{\varepsilon}_{12}^n\,\mathrm{d}x_2}{\mu_m R_{f2}+\mu_f R_{m2}}, \tag{7i}$$

where R_{f2}, R_2, R_3 are as shown in Fig. 2,

$$R_{f2} = \frac{\pi R_f^2}{4R_3}, \quad R_{f3} = \frac{\pi R_f^2}{4R_2},$$

$$R_{m2} = R_2 - R_{f2}, \quad R_{m3} = R_3 - R_{f3}, \tag{8}$$

$$\delta_0 = (2\mu_f+\lambda_f)R_{m3}+(2\mu_m+\lambda_m)R_{f3}-\frac{(\lambda_m R_{f2}+\lambda_f R_{m2})(\lambda_m R_{f3}+\lambda_f R_{m3})}{(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}}, \tag{9}$$

$$A_1 = (\lambda_m-\lambda_f)R_{m2}R_{m3}\left\{1-\frac{\lambda_m R_{f2}+\lambda_f R_{m2}}{(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}}\right\},$$

$$A_2 = R_{m3}\left\{\frac{\lambda_m}{R_{m2}}-\frac{(2\mu_m+\lambda_m)(\lambda_m R_{f2}+\lambda_f R_{m2})}{(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}}\right\},$$

$$A_3 = 2\mu_m+\lambda_m-\frac{\lambda_m(\lambda_m R_{f2}+\lambda_f R_{m2})}{(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}}. \tag{10}$$

For the matrix,

$$\bar{e}_{11}^{nm^+} = \bar{e}_{11}^{nm^-} = \bar{\varepsilon}_{11}^{n}, \tag{11a}$$

$$\bar{e}_{22}^{nm^+} = \frac{1}{R_{m2}}\left\{\int_{x_{02}^m}^{x_{02}^m+R_2}\bar{\varepsilon}_{22}^n\,\mathrm{d}x_2-R_{f2}\bar{e}_{22}^{nf^+}\right\},\quad \bar{e}_{22}^{nm^-} = \frac{1}{R_{m2}}\left\{\int_{x_{02}^m-R_2}^{x_{02}^m}\bar{\varepsilon}_{22}^n\,\mathrm{d}x_2-R_{f2}\bar{e}_{22}^{nf^-}\right\}, \tag{11b}$$

$$\bar{e}_{33}^{nm^+} = \frac{1}{R_{m3}}\left\{\int_{x_{03}^m}^{x_{03}^m+R_3}\bar{\varepsilon}_{33}^n\,\mathrm{d}x_3-R_{f3}\bar{e}_{33}^{nf^+}\right\},\quad \bar{e}_{33}^{nm^-} = \frac{1}{R_{m3}}\left\{\int_{x_{03}^m-R_3}^{x_{03}^m}\bar{\varepsilon}_{33}^n\,\mathrm{d}x_3-R_{f3}\bar{e}_{33}^{nf^-}\right\}, \tag{11c}$$

$$\bar{e}_{23}^{nm^+} = \frac{\mu_f\left\{\int_{x_{02}^m}^{x_{02}^m+R_2}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_2+\frac{1}{2}\int_{x_{03}^m-R_3}^{x_{03}^m+R_3}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_3\right\}}{2\left\{\mu_m R_f+\mu_f\left(\frac{R_2}{R_3}-R_f\right)\right\}},$$

$$\bar{e}_{23}^{nm^-} = \frac{\mu_f\left\{\int_{x_{02}^m-R_2}^{x_{02}^m}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_2+\frac{1}{2}\int_{x_{03}^m-R_3}^{x_{03}^m+R_3}\bar{\varepsilon}_{23}^n\,\mathrm{d}x_3\right\}}{2\left\{\mu_m R_f+\mu_f\left(\frac{R_2}{R_3}-R_f\right)\right\}}, \tag{11d}$$

$$\bar{e}_{31}^{nm^+} = \frac{\mu_f\int_{x_{03}^m}^{x_{03}^m+R_3}\bar{\varepsilon}_{31}^n\,\mathrm{d}x_3}{\mu_m R_{f3}+\mu_f R_{m3}},\quad \bar{e}_{31}^{nm^-} = \frac{\mu_f\int_{x_{03}^m-R_3}^{x_{03}^m}\bar{\varepsilon}_{31}^n\,\mathrm{d}x_3}{\mu_m R_{f3}+\mu_f R_{m3}}, \tag{11e}$$

$$\bar{e}_{12}^{nm^+} = \frac{\mu_f\int_{x_{02}^m}^{x_{02}^m+R_2}\bar{\varepsilon}_{12}^n\,\mathrm{d}x_2}{\mu_m R_{f2}+\mu_f R_{m2}},\quad \bar{e}_{12}^{nm^-} = \frac{\mu_f\int_{x_{02}^m-R_2}^{x_{02}^m}\bar{\varepsilon}_{12}^n\,\mathrm{d}x_2}{\mu_m R_{f2}+\mu_f R_{m2}}. \tag{11f}$$

In the above expressions $\bar{e}_{ij}^{nf^+}(x_{0j}^f)$, $\bar{e}_{ij}^{nf^-}(x_{0i}^f)$, $\bar{e}_{ij}^{nm^+}(x_{0i}^m)$, $\bar{e}_{ij}^{mn^-}(x_{0i}^m)$ indicate the strain values at centers of fiber and matrix regions in transformed coordinate system x_2'- and x_3-directions.

For practical purposes, eqns (7) and (11) can be simplified, therefore calculations are performed in a less complicated and applicable manner. For this reason it can be assumed that the strain components change linearly between the centers of two adjacent material regions and so the in-layer strain distributions become piecewise linear. Following this assumption eqns (7) and (11) for the fibers are reduced to:

$$\bar{e}_{11}^{nf^+} = \bar{e}_{11}^{nf^-} = \bar{\varepsilon}_{11}^{n}, \tag{12a}$$

$$\bar{e}_{22}^{nf^+} = \left\{(\lambda_m-\lambda_f)R_{m2}\bar{\varepsilon}_{11}^n+(2\mu_m+\lambda_m)\frac{R_2}{2}[\bar{\varepsilon}_{22}^n(x_{02}^f+R_2)+\bar{\varepsilon}_{22}^n(x_{02}^f)]\right.$$
$$+\lambda_m\frac{R_{m2}R_3}{2R_{m3}}[\bar{\varepsilon}_{33}^n(x_{03}^f+R_3)+\bar{\varepsilon}_{33}^n(x_{03}^f)]$$
$$\left.-\left(\lambda_m\frac{R_{f3}}{R_{m3}}+\lambda_f\right)R_{m2}\bar{e}_{33}^{nf^+}\right\}\Big/[(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}], \quad (12b)$$

$$\bar{e}_{22}^{nf^-} = \left\{(\lambda_m-\lambda_f)R_{m2}\bar{\varepsilon}_{11}^n+(2\mu_m+\lambda_m)\frac{R_2}{2}[\bar{\varepsilon}_{22}^n(x_{02}^f)+\bar{\varepsilon}_{22}^n(x_{02}^f-R_2)]\right.$$
$$+\lambda_m\frac{R_{m2}R_3}{2R_{m3}}[\bar{\varepsilon}_{33}^n(x_{03}^f)+\bar{\varepsilon}_{33}^n(x_{03}^f-R_3)]$$
$$\left.-\left(\lambda_m\frac{R_{f3}}{R_{m3}}+\lambda_f\right)R_{m2}\bar{e}_{33}^{nf^-}\right\}\Big/[(2\mu_f+\lambda_f)R_{m2}+(2\mu_m+\lambda_m)R_{f2}], \quad (12c)$$

$$\bar{e}_{33}^{nf^+} = \frac{1}{2\delta_0}\{2A_1\bar{\varepsilon}_{11}^n+A_2R_2[\bar{\varepsilon}_{22}^n(x_{02}^f+R_2)+\bar{\varepsilon}_{22}^n(x_{02}^f)]+A_3R_3[\bar{\varepsilon}_{33}^n(x_{03}^f+R_3)+\bar{\varepsilon}_{33}^n(x_{03}^f)]\}, \quad (12d)$$

$$\bar{e}_{33}^{nf^-} = \frac{1}{2\delta_0}\{2A_1\bar{\varepsilon}_{11}^n+A_2R_2[\bar{\varepsilon}_{22}^n(x_{02}^f)+\bar{\varepsilon}_{22}^n(x_{02}^f-R_2)]+A_3R_3[\bar{\varepsilon}_{33}^n(x_{03}^f)+\bar{\varepsilon}_{33}^n(x_{03}^f-R_3)]\}, \quad (12e)$$

$$\bar{e}_{23}^{nf^+} = \frac{\mu_m\{R_2[\bar{\varepsilon}_{23}^n(x_{02}^f+R_2)+\bar{\varepsilon}_{23}^n(x_{02}^f)]+R_3[\bar{\varepsilon}_{23}^n(x_{03}^f+R_3)+\bar{\varepsilon}_{23}^n(x_{03}^f-R_3)]\}}{4\left\{\mu_mR_f+\mu_f\left(\dfrac{R_2+R_3}{2}-R_f\right)\right\}}, \quad (12f)$$

$$\bar{e}_{23}^{nf^-} = \frac{\mu_m\{R_2[\bar{\varepsilon}_{23}^n(x_{02}^f)+\bar{\varepsilon}_{23}^n(x_{02}^f-R_2)]+R_3[\bar{\varepsilon}_{23}^n(x_{03}^f+R_3)+\bar{\varepsilon}_{23}^n(x_{03}^f-R_3)]\}}{4\left\{\mu_mR_f+\mu_f\left(\dfrac{R_2+R_3}{2}-R_f\right)\right\}}, \quad (12g)$$

$$\bar{e}_{31}^{nf^+} = \frac{\mu_mR_3[\bar{\varepsilon}_{31}^n(x_{03}^f+R_3)+\bar{\varepsilon}_{31}^n(x_{03}^f)]}{2(\mu_mR_{f3}+\mu_fR_{m3})}, \quad \bar{e}_{31}^{nf^-} = \frac{\mu_mR_3[\bar{\varepsilon}_{31}^n(x_{03}^f)+\bar{\varepsilon}_{31}^n(x_{03}^f-R_3)]}{2(\mu_mR_{f3}+\mu_fR_{m3})}, \quad (12h)$$

$$\bar{e}_{12}^{nf^+} = \frac{\mu_mR_2[\bar{\varepsilon}_{12}^n(x_{02}^f+R_2)+\bar{\varepsilon}_{121}^n(x_{02}^f)]}{2(\mu_mR_{f2}+\mu_fR_{m2})}, \quad \bar{e}_{12}^{nf^-} = \frac{\mu_mR_2[\bar{\varepsilon}_{12}^n(x_{02}^f)+\bar{\varepsilon}_{12}^n(x_{02}^f-R_2)]}{2(\mu_mR_{f2}+\mu_fR_{m2})}. \quad (12i)$$

For matrix, eqns (7) and (11) are reduced to:

$$\bar{e}_{11}^{nm^+} = \bar{e}_{11}^{nm^-} = \bar{\varepsilon}_{11}^n, \quad (13a)$$

$$\bar{e}_{22}^{nm^+} = \frac{1}{R_{m2}}\left\{\frac{R_2}{2}[\bar{\varepsilon}_{22}^n(x_{02}^m+R_2)+\bar{\varepsilon}_{22}^n(x_{02}^m)]-R_{f2}\bar{e}_{22}^{nf^+}\right\}, \quad (13b)$$

$$\bar{e}_{22}^{nm^-} = \frac{1}{R_{m2}}\left\{\frac{R_2}{2}[\bar{\varepsilon}_{22}^n(x_{02}^m)+\bar{\varepsilon}_{22}^n(x_{02}^m-R_2)]-R_{f2}\bar{e}_{22}^{nf^-}\right\}, \tag{13c}$$

$$\bar{e}_{33}^{nm^+} = \frac{1}{R_{m3}}\left\{\frac{R_3}{2}[\bar{\varepsilon}_{33}^n(x_{03}^m+R_3)+\bar{\varepsilon}_{33}^n(x_{03}^m)]-R_{f3}\bar{e}_{33}^{nf^+}\right\}, \tag{13d}$$

$$\bar{e}_{33}^{nm^-} = \frac{1}{R_{m3}}\left\{\frac{R_3}{2}[\bar{\varepsilon}_{33}^n(x_{03}^m)+\bar{\varepsilon}_{33}^n(x_{03}^m-R_3)]-R_{f3}\bar{e}_{33}^{nf^-}\right\}, \tag{13e}$$

$$\bar{e}_{23}^{nm^+} = \frac{\mu_f\{R_2[\bar{\varepsilon}_{23}^n(x_{02}^m+R_2)+\bar{\varepsilon}_{23}^n(x_{02}^m)]+R_3[\bar{\varepsilon}_{23}^n(x_{03}^m+R_3)+\bar{\varepsilon}_{23}^n(x_{03}^m-R_3)]\}}{4\left\{\mu_m R_f+\mu_f\left(\dfrac{R_2+R_3}{2}-R_f\right)\right\}}, \tag{13f}$$

$$\bar{e}_{23}^{nm^-} = \frac{\mu_f\{R_2[\bar{\varepsilon}_{23}^n(x_{02}^m)+\bar{\varepsilon}_{23}^n(x_{02}^m-R_2)]+R_3[\bar{\varepsilon}_{23}^n(x_{03}^m+R_3)+\bar{\varepsilon}_{23}^n(x_{03}^m-R_3)]\}}{4\left\{\mu_m R_f+\mu_f\left(\dfrac{R_2+R_3}{2}-R_f\right)\right\}}, \tag{13g}$$

$$\bar{e}_{31}^{nm^+} = \frac{\mu_f R_3[\bar{\varepsilon}_{31}^n(x_{03}^m+R_3)+\bar{\varepsilon}_{31}^n(x_{03}^m)]}{2(\mu_m R_{f3}+\mu_f R_{m3})}, \quad \bar{e}_{31}^{nm^-} = \frac{\mu_f R_3[\bar{\varepsilon}_{31}^n(x_{03}^m)+\bar{\varepsilon}_{31}^n(x_{03}^m-R_3)]}{2(\mu_m R_{f3}+\mu_f R_{m3})}, \tag{13h}$$

$$\bar{e}_{12}^{nm^+} = \frac{\mu_f R_2[\bar{\varepsilon}_{12}^n(x_{02}^m+R_2)+\bar{\varepsilon}_{12}^n(x_{02}^m)]}{2(\mu_m R_{f2}+\mu_f R_{m2})}, \quad \bar{e}_{12}^{nm^-} = \frac{\mu_f R_2[\bar{\varepsilon}_{12}^n(x_{02}^m)+\bar{\varepsilon}_{12}^n(x_{02}^m-R_2)]}{2(\mu_m R_{f2}+\mu_f R_{m2})}. \tag{13i}$$

The actual stresses in the fibers and the matrix are affected by all the neighboring material regions. The interaction effects are due to the heterogeneity and therefore act only in the directions perpendicular to the axis of the fibers. In this study, since there is only one row of fiber bundles in each lamina, the second and higher order heterogeneity effects in the direction perpendicular to the layer plane are ignored, that is the x_3-direction in Fig. 1. Therefore, for the composite structure shown in Fig. 1, there exist second and higher order heterogeneity effects only in the x_2'-direction (x_1' is the reinforcement direction). Numerical calculations show that if the gradient of far-field strains is not extremely high, then the third and higher order heterogeneity effects are insignificant. But in this case since a high strain gradient problem is considered the higher order heterogeneity effects must be taken into consideration. In the case of linear or uniform strain distributions the interactions between the adjacent material regions are the same, but in the case of nonlinear strain distribution these interactions will be different everywhere in the composite body, therefore the expressions appear more complicated. Following these statements, the resultant strain components in the fibers and the matrix of layer "n" are obtained as follows:

for the fibers

$$\bar{\varepsilon}_{11}^{nf} = \bar{\varepsilon}_{11}^n+S_{11}^{nf}, \quad \bar{\varepsilon}_{22}^{nf} = \bar{\varepsilon}_{22}^n+S_{22}^{nf}, \quad \bar{\varepsilon}_{33}^{nf} = \bar{\varepsilon}_{33}^n+S_{33}^{nf},$$
$$\bar{\varepsilon}_{23}^{nf} = \bar{\varepsilon}_{23}^n+S_{23}^{nf}, \quad \bar{\varepsilon}_{31}^{nf} = \bar{\varepsilon}_{31}^n+S_{31}^{nf}, \quad \bar{\varepsilon}_{12}^{nf} = \bar{\varepsilon}_{12}^n+S_{12}^{nf}, \tag{14}$$

for the matrix

$$\bar{\varepsilon}_{11}^{nm} = \bar{\varepsilon}_{11}^n+S_{11}^{nm}, \quad \bar{\varepsilon}_{22}^{nm} = \bar{\varepsilon}_{22}^n+S_{22}^{nm}, \quad \bar{\varepsilon}_{33}^{nm} = \bar{\varepsilon}_{33}^n+S_{33}^{nm},$$
$$\bar{\varepsilon}_{23}^{nm} = \bar{\varepsilon}_{23}^n+S_{23}^{nm}, \quad \bar{\varepsilon}_{31}^{nm} = \bar{\varepsilon}_{31}^n+S_{31}^{nm}, \quad \bar{\varepsilon}_{12}^{nm} = \bar{\varepsilon}_{12}^n+S_{12}^{nm}. \tag{15}$$

In eqns (14) and (15) S_{ij}^{nf} and S_{ij}^{nm} are interaction terms which can be expressed as follows:

$$S_{11}^{nf} = S_{11}^{nm} = 0. \tag{16}$$

For "22", "23" and "12" components

$$S_{ij}^{nf} = \frac{1}{2}s_{ij}^{nf^-} f_{ij}^{n^-}(x_{02}^f)[1 - m_{ij}^n(x_{02}^f - R_2)]$$

$$+ \frac{1}{2}c_{ij}^f s_{ij}^{nf^-} \sum_{-\infty}^{l=-2} f_{ij}^{n^-}(x_{02}^f) \prod_{l}^{k_f=-2} f_{ij}^n(x_{02}^f + (2k_f+1)R_2)$$

$$\times \left\{ \prod_{l-1}^{k_m=-1} m_{ij}^n(x_{02}^f + (2k_m+1)R_2) - \prod_{l}^{k_m=-1} m_{ij}^n(x_{02}^f + (2k_m+1)R_2) \right\}$$

$$+ \frac{1}{2}s_{ij}^{nf^+} f_{ij}^{n^+}(x_{02}^f)[1 - m_{ij}^n(x_{02}^f + R_2)]$$

$$+ \frac{1}{2}c_{ij}^f s_{ij}^{nf^+} \sum_{l=2}^{\infty} f_{ij}^{n^+}(x_{02}^f) \prod_{k_f=2}^{l} f_{ij}^n(x_{02}^f + (2k_f-1)R_2)$$

$$\times \left\{ \prod_{k_m=1}^{l-1} m_{ij}^n(x_{02}^f + (2k_m-1)R_2) - \prod_{k_m=1}^{l} m_{ij}^n(x_{02}^f + (2k_m-1)R_2) \right\} \tag{17a}$$

$$S_{ij}^{nm} = \frac{1}{2}s_{ij}^{nm^-} m_{ij}^{n^-}(x_{02}^m)[1 - f_{ij}^n(x_{02}^m - R_2)]$$

$$+ \frac{1}{2}c_{ij}^m s_{ij}^{nm^-} \sum_{-\infty}^{l=-2} m_{ij}^{n^-}(x_{02}^m) \prod_{l}^{k_m=-2} m_{ij}^n(x_{02}^m + (2k_m+1)R_2)$$

$$\times \left\{ \prod_{l-1}^{k_f=-1} f_{ij}^n(x_{02}^m + (2k_f+1)R_2) - \prod_{l}^{k_f=-1} f_{ij}^n(x_{02}^m + (2k_f+1)R_2) \right\}$$

$$+ \frac{1}{2}s_{ij}^{nm^+} m_{ij}^{n^+}(x_{02}^m)[1 - f_{ij}^n(x_{02}^m + R_2)]$$

$$+ \frac{1}{2}c_{ij}^m s_{ij}^{nm^+} \sum_{l=2}^{\infty} m_{ij}^{n^+}(x_{02}^m) \prod_{k_m=2}^{l} m_{ij}^n(x_{02}^m + (2k_m-1)R_2)$$

$$\times \left\{ \prod_{k_f=1}^{l-1} f_{ij}^n(x_{02}^m + (2k_f-1)R_2) - \prod_{k_f=1}^{l} f_{ij}^n(x_{02}^m + (2k_f-1)R_2) \right\}. \tag{17b}$$

For "33" and "31" components

$$S_{ij}^{nf} = s_{ij}^{nf} f_{ij}^n, \quad S_{ij}^{nm} = s_{ij}^{nm} m_{ij}^n, \tag{18}$$

where there is no summation over the repeated indices. Superscript "n" indicates the layer whose fibers and matrix are considered. f_{ij}^n and m_{ij}^n are the average adjacent material region effects, s_{ij}^{nf} and s_{ij}^{nm} are heterogeneity effect sign terms which can be expressed as

$$f_{ij}^{n+} = |\bar{\varepsilon}_{ij}^{nf^+} - \bar{\varepsilon}_{ij}^n|, \quad f_{ij}^{n-} = |\bar{\varepsilon}_{ij}^{nf^-} - \bar{\varepsilon}_{ij}^n|, \tag{19a}$$

$$m_{ij}^{n+} = |\bar{\varepsilon}_{ij}^{nm^+} - \bar{\varepsilon}_{ij}^n|, \quad m_{ij}^{n-} = |\bar{\varepsilon}_{ij}^{nm^-} - \bar{\varepsilon}_{ij}^n|, \tag{19b}$$

$$f_{ij}^n = \tfrac{1}{2}(f_{ij}^{n+} + f_{ij}^{n-}), \quad m_{ij}^n = \tfrac{1}{2}(m_{ij}^{n+} + m_{ij}^{n-}), \tag{19c}$$

$$s_{ij}^{nf^+} = \operatorname{sgn}(\bar{\varepsilon}_{ij}^{nf^+} - \bar{\varepsilon}_{ij}^n), \quad s_{ij}^{nf^-} = \operatorname{sgn}(\bar{\varepsilon}_{ij}^{nf^-} - \bar{\varepsilon}_{ij}^n), \tag{20a}$$

$$s_{ij}^{nm^+} = \operatorname{sgn}(\bar{\varepsilon}_{ij}^{nm^+} - \bar{\varepsilon}_{ij}^n), \quad s_{ij}^{nm^-} = \operatorname{sgn}(\bar{\varepsilon}_{ij}^{nm^-} - \bar{\varepsilon}_{ij}^n). \tag{20b}$$

In eqns (17) c_{ij}^f and c_{ij}^m terms are functions that give the variations of the heterogeneity effects within the different material regions. These functions can be found by interpolation of the interaction effects from different regions on the basis of the proximity of the adjacent regions to a particular point. The resulting functions will be continuous within a material region and satisfy matching conditions at the interfaces between regions. These simple functions have been explained by Ardıç *et al.* (1990) and their expressions are as follows:

$$c_{ij}^f = \frac{15}{8}\sqrt{\frac{R_f - r_f}{R_f}},$$

$$c_{22}^m = c_{12}^m = \frac{3}{2}\sqrt{\frac{R_{m2} - r_{m2}}{R_{m2}}},$$

$$c_{23}^m = 3(R_2 + R_3 - R_f)\frac{\sqrt{R_{m2} - r_{m2}} + \sqrt{R_{m3} - r_{m3}}}{2(R_{m2}^{3/2} + R_{m3}^{3/2})}, \tag{21}$$

where R_{m2} and R_{m3} are as given before, r_{m2}, r_{m3} and r_f are shown in Fig. 2.

The strain fields expressed in eqns (14) and (15) are relative to the in-layer coordinate system in which the fiber direction is x_1'-direction. These strain fields should be expressed in the original global coordinate system, therefore the strain components are transformed back. These strains in the original global coordinate system are represented with ε_{ij}^{nf} and ε_{ij}^{nm}. By using these strain components the stress fields in the fibers and the matrix, in the original coordinate system can be expressed for the fibers as

$$\tau_{ij}^{nf} = 2\mu_f \varepsilon_{ij}^{nf} + \lambda_f \delta_{ij} \varepsilon_{kk}^{nf}, \tag{22}$$

and for the matrix as

$$\tau_{ij}^{nm} = 2\mu_m \varepsilon_{ij}^{nm} + \lambda_m \delta_{ij} \varepsilon_{kk}^{nm}. \tag{23}$$

Therefore, a way of expressing the stress and strain fields in the fibers and the matrix of any lamina in a laminated composite body under any kind of loading is obtained. By using the strain and stress values calculated from eqns (14)–(23), a stress and strain analysis can be performed.

4. SAMPLE RESULTS AND DISCUSSIONS

In the previous sections the strain fields in each layer have been determined, and used as input strains for the calculations of fiber and matrix strain fields separately. By using the fiber and matrix strains the stresses in the fiber and the matrix can be computed. With the method developed herein a detailed strain and stress analysis of any laminated composite subject to high stress and strain gradients can be performed.

As an example for the case of nonlinear stress and strain distributions with high gradients a laminated composite plate with a hole in the center is taken. Figure 3 depicts the one quarter of the plate which is loaded in tension by the application of a concentrated force of 10 kN. The complete dimensions of the plate are 200 mm × 200 mm and 1.6 mm in thickness. The material used is ICI Fiberite T300/934 carbon epoxy [ICI Fiberite (1989)]. Fiber volume fraction is taken as 60% and fiber and matrix properties are $E_f = 228$ GPa, $\nu_f = 0.255$, $E_m = 4.1$ GPa and $\nu_m = 0.33$, respectively. The sample plate is made of 16 plies of equal thickness with a stacking sequence of $[0°/45°/-45°/90°/0°/45°/-45°/90°]_s$.

It should be noted that the existence of a hole in the middle of the plate accounts for high stress and high strain gradient, due to the stress concentration effect in the immediate vicinity of the hole edge. The input far-field strains are calculated by means of MSC/Nastran finite element software using the eight node solid elements. In the finite element model only a single layer is used and the material is assigned orthotropic properties representing the

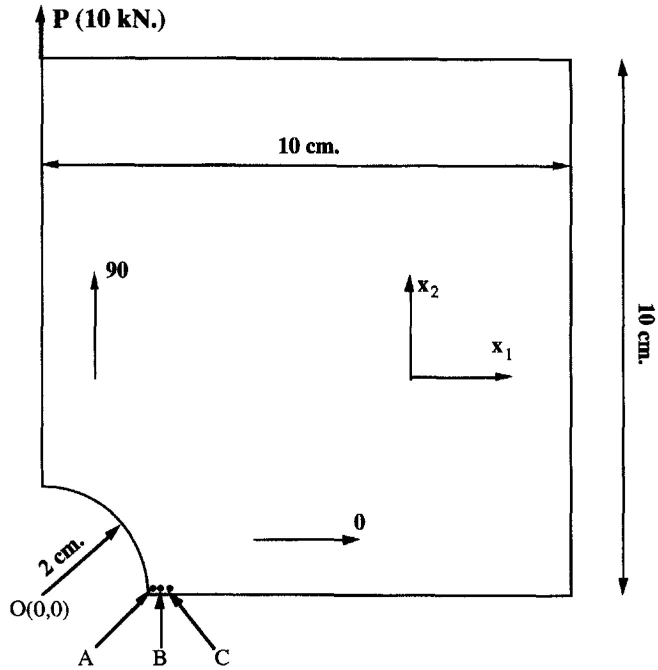

Fig. 3. Dimensions of and loading on the sample composite.

above mentioned stacking sequence. As it is shown in Fig. 3, the input far-field strains are calculated at three locations, A, B, and C which are at a distance of 20.5 mm, 21.5 mm, and 22.5 mm from the center of the hole, respectively. This way the analysis can be performed at different points in the high strain gradient region. The input strains calculated using the eight node solid element model are summarized in Table 1. By using the input strains in the expressions given by eqns (14) and (15), fiber and matrix strains at locations A, B, and C are determined separately. These fiber and matrix strains are compared with the MSC/Nastran composite layer solution (shell elements with PCOMP properties) in Table 2. In the Nastran composite layer solution the same plate is modeled with shell elements, and strains in each ply are determined. However, since Nastran composite model uses shell elements only, its output does not include strains with subscripts "3", i.e. $e_{3i} = 0$ ($i = 1, 2, 3$). On the other hand, the method of this study produces strains in the thickness direction, e_{3i} in general and e_{33} for this sample case, although no input strain was given in that direction. As a better verification way of the second stage of the non-classical method which is the calculation of fiber and matrix strains from in-layer strains, MSC/Nastran layered composite solution results are used as in-layer strains. Results of the non-classical method and Nastran results are compared in Table 3. All of the results in Table 3 show that Nastran layer strains are between the matrix and fiber strains calculated by using the method presented in this study. This is a very reasonable result and verifies the method used herein. Some of the results in Table 2 show that Nastran layer strains do not fall between matrix and fiber strains, this is because the Nastran solution is two-dimensional and the non-classical solution is three-dimensional. Therefore, Nastran solution should

Table 1. Input far-field strains

	e_{11}	e_{22}	e_{33}	e_{23}	e_{31}	e_{12}
A	-3.95×10^{-3}	5.17×10^{-3}	0	0	0	1.41×10^{-5}
B	-3.26×10^{-3}	4.41×10^{-3}	0	0	0	-1.43×10^{-4}
C	-2.79×10^{-3}	3.86×10^{-3}	0	0	0	-9.75×10^{-5}

Table 2. Strains from the non-classical method and MSC/Nastran layered composite solution

	Layers	A Nastran	A Fiber	A Matrix	B Nastran	B Fiber	B Matrix	C Nastran	C Fiber	C Matrix
e_{11}	0	-2.047×10^{-3}	-3.959×10^{-3}	-3.959×10^{-3}	-1.525×10^{-3}	-3.265×10^{-3}	-3.265×10^{-3}	-1.137×10^{-3}	-2.798×10^{-3}	-2.798×10^{-3}
	45	-6.708×10^{-3}	1.8×10^{-5}	-9.938×10^{-3}	-5.475×10^{-3}	-9.2×10^{-6}	-8.158×10^{-3}	-4.532×10^{-3}	1.6×10^{-5}	-7.026×10^{-3}
	-45	-6.708×10^{-3}	9.4×10^{-6}	-9.924×10^{-3}	-5.475×10^{-3}	8.6×10^{-5}	-8.301×10^{-3}	-4.532×10^{-3}	8.1×10^{-5}	-7.123×10^{-3}
	90	-2.047×10^{-3}	-1.691×10^{-3}	-7.364×10^{-3}	-1.525×10^{-3}	-1.434×10^{-3}	-6.014×10^{-3}	-1.138×10^{-3}	-1.251×10^{-3}	-5.118×10^{-3}
e_{22}	0	7.276×10^{-3}	1.410×10^{-3}	1.0842×10^{-2}	6.374×10^{-3}	1.168×10^{-3}	9.282×10^{-3}	5.652×10^{-3}	1.003×10^{-3}	8.153×10^{-3}
	45	1.1937×10^{-2}	4.54×10^{-4}	1.2280×10^{-2}	1.0324×10^{-2}	3.48×10^{-4}	1.0513×10^{-2}	9.047×10^{-3}	3.22×10^{-4}	9.176×10^{-3}
	-45	1.1937×10^{-2}	4.45×10^{-4}	1.2294×10^{-2}	1.0324×10^{-2}	4.44×10^{-4}	1.0370×10^{-2}	9.047×10^{-3}	3.87×10^{-4}	9.079×10^{-3}
	90	7.276×10^{-3}	5.179×10^{-3}	5.179×10^{-3}	6.374×10^{-3}	4.410×10^{-3}	4.410×10^{-3}	5.652×10^{-3}	3.861×10^{-3}	3.861×10^{-3}
e_{33}	0	N.A.	5.57×10^{-4}	2.541×10^{-3}	N.A.	4.63×10^{-4}	2.141×10^{-3}	N.A.	3.99×10^{-4}	1.861×10^{-3}
	45	N.A.	-4.8×10^{-5}	7.0×10^{-5}	N.A.	-2.5×10^{-5}	3.8×10^{-5}	N.A.	-2.8×10^{-5}	4.2×10^{-5}
	-45	N.A.	-4.3×10^{-5}	6.5×10^{-5}	N.A.	-5.9×10^{-5}	8.8×10^{-5}	N.A.	-5.1×10^{-5}	7.6×10^{-5}
	90	N.A.	-6.47×10^{-4}	-2.404×10^{-3}	N.A.	-5.47×10^{-4}	-2.013×10^{-3}	N.A.	-4.77×10^{-4}	-1.743×10^{-3}
e_{12}	0	-1.6×10^{-5}	5.8×10^{-7}	3.4×10^{-5}	1.29×10^{-4}	-6.0×10^{-6}	-3.50×10^{-4}	1.83×10^{-4}	-4.0×10^{-6}	-2.37×10^{-4}
	45	-8.2×10^{-6}	3.88×10^{-4}	-5.47×10^{-4}	6.5×10^{-5}	2.59×10^{-4}	-7.48×10^{-4}	9.1×10^{-5}	2.64×10^{-4}	-6.41×10^{-4}
	-45	-8.2×10^{-6}	-3.69×10^{-4}	5.89×10^{-4}	6.5×10^{-5}	-4.51×10^{-4}	3.17×10^{-4}	9.1×10^{-5}	-3.94×10^{-4}	3.48×10^{-4}
	90	-1.6×10^{-5}	5.8×10^{-7}	3.4×10^{-5}	1.29×10^{-4}	-5.94×10^{-6}	-3.49×10^{-4}	1.83×10^{-4}	-4.0×10^{-6}	-2.37×10^{-4}

N.A. = not applicable.

Table 3. Comparison of MSC/Nastran layered composite solution and non-classical method results using Nastran output as in-layer strains

	Layers	A Nastran	A Fiber	A Matrix	B Nastran	B Fiber	B Matrix	C Nastran	C Fiber	C Matrix
e_{11}	0	-2.047×10^{-3}	-2.047×10^{-3}	-2.047×10^{-3}	-1.525×10^{-3}	-1.525×10^{-3}	-1.525×10^{-3}	-1.137×10^{-3}	-1.137×10^{-3}	-1.137×10^{-3}
	45	-6.708×10^{-3}	7.71×10^{-4}	-6.842×10^{-3}	-5.475×10^{-3}	7.84×10^{-4}	-5.509×10^{-3}	-4.532×10^{-3}	7.66×10^{-4}	-4.696×10^{-3}
	-45	-6.708×10^{-3}	7.81×10^{-4}	-6.800×10^{-3}	-5.475×10^{-3}	6.98×10^{-4}	-5.499×10^{-3}	-4.532×10^{-3}	6.45×10^{-4}	-4.614×10^{-3}
	90	-2.047×10^{-3}	-2.200×10^{-3}	-1.818×10^{-3}	-1.525×10^{-3}	-1.914×10^{-3}	-9.41×10^{-4}	-1.138×10^{-3}	-1.687×10^{-3}	-3.12×10^{-4}
e_{22}	0	7.276×10^{-3}	9.4×10^{-4}	1.6727×10^{-2}	6.374×10^{-3}	7.88×10^{-4}	1.4774×10^{-2}	5.652×10^{-3}	6.33×10^{-4}	1.3198×10^{-2}
	45	1.1937×10^{-2}	1.216×10^{-3}	1.6383×10^{-2}	1.0324×10^{-2}	1.153×10^{-4}	1.4218×10^{-2}	9.047×10^{-3}	1.078×10^{-3}	1.2522×10^{-2}
	-45	1.1937×10^{-2}	1.227×10^{-3}	1.6367×10^{-2}	1.0324×10^{-2}	1.068×10^{-3}	1.4346×10^{-2}	9.047×10^{-3}	9.57×10^{-4}	1.2704×10^{-2}
	90	7.276×10^{-3}	7.276×10^{-3}	7.276×10^{-3}	6.374×10^{-3}	6.374×10^{-3}	6.374×10^{-3}	5.652×10^{-3}	5.652×10^{-3}	5.652×10^{-3}
e_{33}	0	N.A.	4.21×10^{-4}	2.813×10^{-3}	N.A.	3.41×10^{-4}	2.407×10^{-3}	N.A.	2.80×10^{-4}	2.088×10^{-3}
	45	N.A.	-1.91×10^{-4}	2.87×10^{-4}	N.A.	-1.94×10^{-4}	2.91×10^{-4}	N.A.	-1.88×10^{-4}	2.82×10^{-4}
	-45	N.A.	-1.95×10^{-4}	2.92×10^{-4}	N.A.	-1.64×10^{-4}	2.46×10^{-4}	N.A.	-1.45×10^{-4}	2.18×10^{-4}
	90	N.A.	-8.07×10^{-4}	-2.232×10^{-3}	N.A.	-6.99×10^{-4}	-1.868×10^{-3}	N.A.	-6.13×10^{-4}	-1.587×10^{-3}
e_{12}	0	-1.6×10^{-5}	-6.6×10^{-7}	-3.9×10^{-5}	1.29×10^{-4}	5.4×10^{-6}	3.14×10^{-4}	1.83×10^{-4}	7.6×10^{-6}	4.46×10^{-4}
	45	-8.2×10^{-6}	1.605×10^{-3}	-2.451×10^{-3}	6.5×10^{-5}	1.585×10^{-3}	-2.058×10^{-3}	9.1×10^{-5}	1.519×10^{-3}	-1.823×10^{-3}
	-45	-8.2×10^{-6}	-1.626×10^{-3}	2.403×10^{-4}	6.5×10^{-5}	-1.412×10^{-3}	2.444×10^{-3}	9.1×10^{-5}	-1.273×10^{-3}	2.370×10^{-3}
	90	-1.6×10^{-5}	-6.6×10^{-7}	-3.9×10^{-5}	1.29×10^{-4}	5.4×10^{-6}	3.14×10^{-4}	1.83×10^{-4}	7.6×10^{-6}	4.46×10^{-4}

N.A. = not applicable.

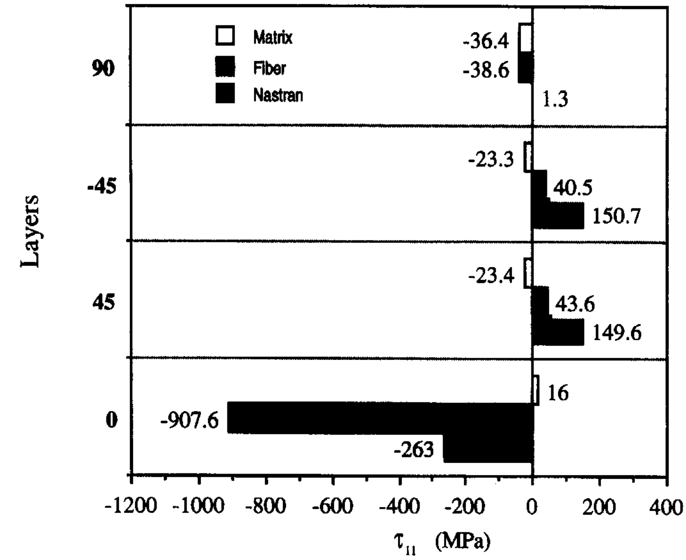

Fig. 4. Comparison of stress component τ_{11}.

satisfy two-dimensional compatibility and the non-classical solution should satisfy three-dimensional compatibility by taking the effect of "33" component into account. Another reason is the calculation of input far-field strains. Nastran composite solution is used to find the strains in each layer, but the far-field strains are calculated by using Nastran anisotropic material solution with solid elements and these two solutions give different results.

Figures 4, 5, 6 and 7 in which the data in Table 2 are used, give the stress components τ_{11}, τ_{22}, τ_{12} and τ_{33} in units of MPa, respectively. The fiber and matrix stresses are compared with the MSC/Nastran composite layer stress output. In these figures the Nastran layer solution does not come out as the average of fiber and matrix stresses. The reason for this discrepancy is due to the fact that Nastran composite solution uses the classical approach of rule of mixtures whereas the method used here does not necessitate the calculation of an effective moduli of the laminate. Material moduli of the fibers and the matrix are utilized separately in the non-classical approach. Furthermore, since the Nastran solution is two-dimensional it is not very accurate. In Fig. 7 no Nastran solution is shown for τ_{33} because

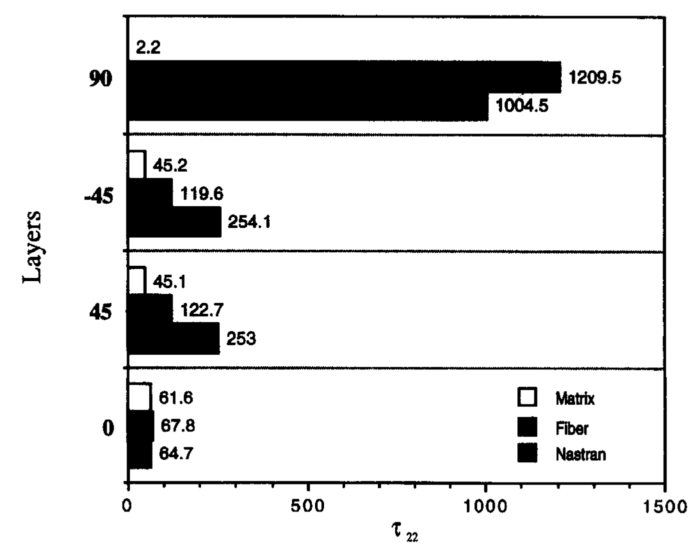

Fig. 5. Comparison of stress component τ_{22}.

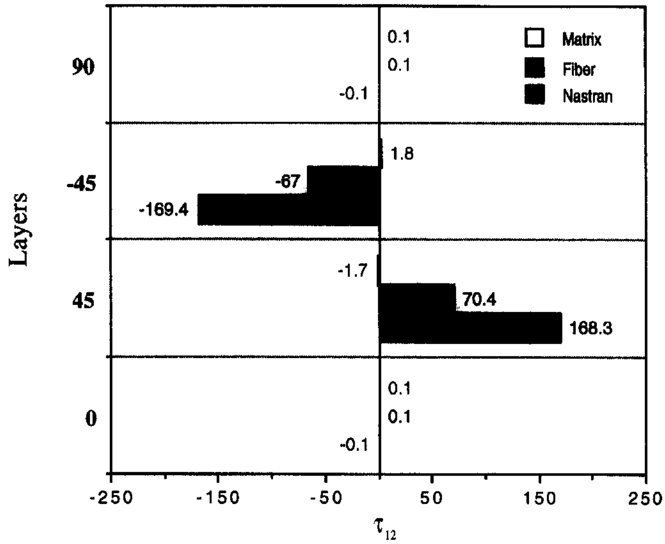

Fig. 6. Comparison of stress component τ_{12}.

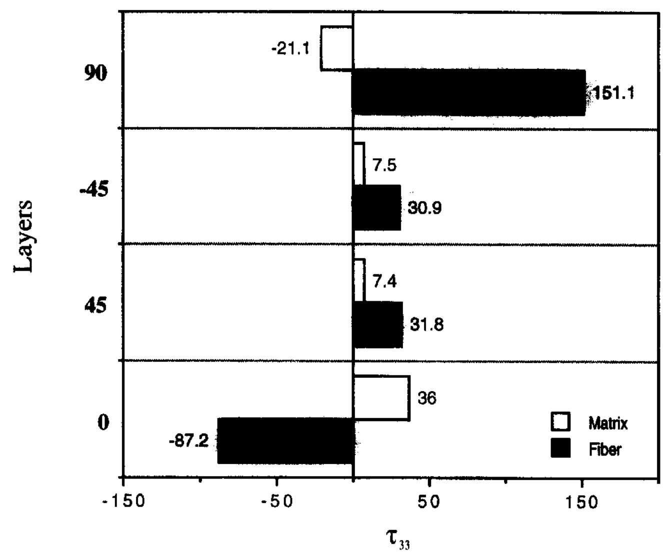

Fig. 7. Fiber and matrix stress components τ_{33}.

as in the case of strains Nastran composite solution does not produce stresses in the thickness direction.

It was already mentioned that in calculating the fiber and matrix strains, the strains at the center of the Nastran solid elements were taken as the input. In the solid model the aspect ratio of the elements was somewhat high and therefore the input strains may be a rough input for the calculation of fiber and matrix strains. The discrepancy between the Nastran composite solution and method of the present study might also be due to the rough input strain values extracted from the high aspect ratio solid finite element model. With a finer mesh in the high stress–strain gradient region more reliable input strain information can be obtained. In the present study these input values are used to demonstrate the ability of non-classical method in the calculation of layer strains as well as fiber and matrix strains.

In general, the input strains in this method do not necessarily have to be calculated by using a finite element method, but classical analytical methods for homogeneous materials can also be used. If the expressions of displacements or strains are known they can be used in the equations of layer, fiber and matrix strains in the non-classical method as input. It should also be stated that even though the results of the sample problem are solved for the piecewise linear input strain case, in general any nonlinear strain distribution can be used as input for the determination of fiber and matrix strains.

5. SUMMARY AND CONCLUDING REMARKS

In this study, a method was developed to determine the stress and strain fields in the different materials of a laminated composite. In this method the distribution of strains was considered to be the most general case, namely, nonlinear and high gradient. Local stresses are calculated from input far-field strains for the layers and from in-layer strains for the fibers and the matrix. Equations were derived, first, for the layer heterogeneity effects then for the fiber and the matrix long-range interactions. Therefore, a way of expressing the strains in the fibers and the matrix of the composite body was found for a very general case.

The significance of this method is that it can be used to predict failure initiation sites for the problems which can be solved by using other methods like finite element, classical elasticity, etc. Since this method was developed for nonlinear stress and strain distributions with high gradients, it can easily be applied to any problem like the ones having linear or uniform strain and stress distributions. Additionally, the numerical results show that by using this method more detailed information can be obtained about the stress and strain distributions in the laminated composites. Therefore, in an engineering point of view, the method developed in this study provides a new tool for more reliable designs.

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